Kinematically Constrained Workspace Control via Linear Optimization*

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Abstract—We present a method for Cartesian workspace control of a robot manipulator that enforces joint-level acceleration, velocity, and position constraints using linear optimization. This method is robust to kinematic singularities. On redundant manipulators, we avoid poor configurations near joint limits by including a maximum permissible velocity term to center each joint within its limits. Compared to the baseline Jacobian damped least-squares method of workspace control, this new approach honors kinematic limits, ensuring physically realizable control inputs and providing smoother motion of the robot. We demonstrate our method on simulated redundant and non-redundant manipulators and implement it on the physical 7-degree-of-freedom Baxter manipulator. We provide our control software under a permissive license.

I. INTRODUCTION

Many manipulation tasks require direct motion in the robot’s Cartesian workspace. Examples of such tasks include servoing an end-effector towards a target object, maintaining a workspace constraint such as the rotation axis of a valve, or tracking a trajectory through workspace waypoints. The conventional solution for direct workspace motion uses the manipulator Jacobian to map from desired workspace motions to joint-level commands for the robot. Jacobian inverse kinematics (IK) is efficient, effective, and widely implemented. However, Jacobian IK does not directly account for joint-level constraints on the kinematics of the arm, such as position limits, maximum achievable velocities, or desired acceleration limits for smooth motion. Additionally, it is common to execute lower-priority tasks by projecting the desired task velocity into the nullspace of the higher-priority manipulator Jacobian; if not treated carefully this can cause large accelerations at the start of motion. To address these challenges of classic Jacobian IK, we present a new workspace control method that uses linear programming to find an optimal, achievable solution to tracking a workspace reference while respecting the kinematic constraints of the robot.

The proposed linearly-constrained Cartesian control (LC³) method uses linear programming to compute locally optimal, achievable workspace motions that satisfy joint-level constraints. The linear program computes the achievable, instantaneous workspace acceleration that will best track the desired velocity for the current time step (see section II). The linear constraints on the system are found in terms of the change in joint velocity, incorporating the Jacobian pseudo-inverse into the constraint matrix to map from workspace to joint-space values (see subsection II-B). In addition, a scaled nullspace velocity is also included. The solution to the linear program is a workspace acceleration which respects the kinematic constraints on the system and is singularity robust by way of using the damped pseudo-inverse of the Jacobian. We implement LC³ and demonstrate results in simulation and on physical hardware (see section III). We provide our software under a permissive license. ²

A. Related Work

Jacobian IK for robotics is a well studied approach [6], [21]. Jacobian least-squares methods compute to numerical precision the joint velocities to match a workspace reference. Singularity-robust Jacobian damped least squares methods operate reliably near singular configurations of the manipulator [18], [5]. Jacobian-based methods are also used to execute hierarchies of workspaces tasks [19]. Jacobian methods, however, do not directly account for joint-level constraints of the manipulator, e.g., acceleration limits, thus requiring additional considerations – or assumptions on initial state – to produce feasible motion of the manipulator. The LC³ method proposed in this paper builds on Jacobian inverse

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² Software available at http://github.com/golems/reflex
kinematics to directly address joint-level constraints through linear programming.

Position-based IK methods are distinct from Jacobian-based derivative methods, though one can solve position IK via gradient descent using the Jacobian. Analytical inverse kinematic solutions have been found for certain manipulator topologies, e.g., for anthropomorphic limbs [24]. Cyclic coordinate descent (CCD) effectively finds position IK solutions for arbitrary manipulators [25]. The FABRIK method by Aristidou and Lasenby uses a heuristic approach to find efficient IK solutions for chains of rotational joints [1]. Position IK methods find a joint-space position that achieves a desired workspace position. Respecting velocity and acceleration constraints while servoing to the computed joint-space position requires additional considerations or assumptions. In contrast, LC$^3$ finds a feasible joint-space velocity that best achieves a desired workspace velocity, directly accounting for joint-level constraints.

Quadratic optimization has also been applied to methods of robot control. Hierarchical quadratic programs can be used to satisfy multiple workspace tasks [9], [11]. Kim and Oh present a quadratic programming framework for position control that respects position, velocity, and acceleration limits [16]. Team MIT used sequential quadratic programs to find constrained IK solutions for an Atlas humanoid robot [10]. [12] uses quadratic programming to find optimal control to find constrained IK solutions for an Atlas humanoid robot limits [16]. T eam MIT used sequential quadratic programs used to satisfy multiple workspace tasks [9], [11]. Kim and directly accounting for joint-level constraints.

The robot is modeled as an $n$-degree-of-freedom (DOF) manipulator operating in a workspace of dimension $m$. We represent a manipulator joint configuration with variable $q$. Workspace positions are designated with variable $X$. Time derivatives of some variable $y$ are represented as $\dot{y}$, and double time derivatives as $\ddot{y}$.

We denote actual, reference, and constraint variables with the following subscripts. Actual joint configurations observed on the robot are designated with subscript $a$. We assume that that actual joint position $q_a$ and actual joint velocity $\dot{q}_a$ are observable. Desired reference values of a trajectory are given by subscript $r$. For a workspace motion, we have the desired workspace position $X_r$ and desired workspace velocity $\dot{X}_r$. Control commands are designated by subscript $u$, e.g., the controlled joint velocity $\dot{q}_u$. Constraints on the system are designated by bounding constants with subscript $\min$ and $\max$ for the minimum and maximum values on the system respectively.

The manipulator Jacobian matrix $J$ and its pseudo-inverse $J^*$ relate joint-space and workspace derivatives:

$$\dot{X} = J\dot{q}$$

$$\dot{q} = J^*\dot{X}$$

The damped Jacobian pseudo-inverse $J^+$ produces acceptable motion near kinematic singularities, avoiding large velocities produced by the undamped pseudo-inverse $J^*$. The damped pseudo-inverse can be computed as follows [18].

$$J^+ = J^T(JJ^T + \lambda^2I)^{-1}$$

where $\lambda$ is the damping constant and $I$ is the identity matrix.

We can also use the singular value decomposition (SVD) of the Jacobian $J$ to damp the solution only near singularities, avoiding error from damping when far from singularities:

$$J = USV^T$$

$$J^+ = \sum_{i=1}^{\min(m,n)} \frac{s_i}{\max(s_i^2, s_r^2)} v_i u_i^T$$

where $s_r$ is a threshold on the singular values below which we introduce damping.

The nullspace of the manipulator Jacobian represents joint velocities which will not change the end-effector velocities. We project a desired joint space velocity into the Jacobian nullspace using projection matrix $N$:

$$N = I - J^+J$$

$$\dot{q}_n = N\dot{q}_r$$

where $\dot{q}_r$ is a desired joint velocity and $\dot{q}_n$ is the projection into the Jacobian nullspace.

In our case, we project velocities to move joints away from position limits. This velocity is designated by $\dot{q}_{rn}$ (see subsection II-B).

The standard form of a Linear Program (LP) is:

Maximize $c^T x$

Subject to $Ax \leq b$

and $x > 0$  (1)

where $x$ is the vector of optimization variables, $A$ and $b$ encode the linear constraints, and $c$ encodes the linear objective function. LC$^3$ in standard LP form is discussed in subsection II-D.
II. Method

Our proposed linearly-constrained Cartesian control (LC\textsuperscript{3}) uses linear optimization to compute the joint-space commands that best track a workspace reference, subject to position, acceleration, and velocity constraints. LC\textsuperscript{3} is given actual configuration \(q_a\) and velocity \(\dot{q}_a\), manipulator Jacobian \(J\), desired workspace velocity \(\dot{X}_r\), and desired joint velocity \(\dot{q}_{rn}\) (for nullspace projection). The robot motion is constrained based on joint-space limits in position \(q_{min}\) and \(q_{max}\), velocity \(\dot{q}_{min}\) and \(\dot{q}_{max}\), and acceleration \(\ddot{q}_{min}\) and \(\ddot{q}_{max}\). The output of LC\textsuperscript{3} is a feasible joint-space velocity command \(\dot{q}_a\) that is within the specified constraints and that optimally matches \(\dot{X}_r\) and \(\dot{q}_{rn}\). We assume that the robot provides joint-level velocity control.

A. Objective Function

The optimization variables for LC\textsuperscript{3} are achievable workspace accelerations \(\dot{X}_u\) and a nullspace projection gain \(k\). We optimize over workspace accelerations to enable feedback correction of position and velocity errors directly in workspace. By optimizing workspace acceleration \(\dot{X}_u\), we can define a linear objective function that most closely matches \(\dot{X}_u\) with the necessary acceleration to achieve the desired velocity \(\dot{X}_r\). The computed optimization variable \(\dot{X}_u\) will accelerate as closely as possible to the desired velocity \(\dot{X}_r\) for the current timestep. We address the signs of optimization variables in subsection II-C via a sign transformation.

We compute a desired workspace acceleration \(\dot{X}_r\) based on the desired velocity vector in workspace \(\dot{X}_r\), and the robot’s actual velocity \(\dot{q}_a\):

\[
\dot{X}_a = J\dot{q}_a \\
\dot{X}_r = \frac{\dot{X}_a - \dot{X}_r}{\Delta t}
\]

where \(\dot{X}_a\) is the actual workspace velocity and \(\Delta t\) is the control time step.

The linear objective function to find the optimal acceleration \(\dot{X}_u\) is:

\[
\dot{X}_r \cdot \dot{X}_u + C_u k
\]

We use the dot product between the desired and controlled accelerations, \(\dot{X}_r \cdot \dot{X}_u\), because it relates to the directions of these vectors. If the acceleration vectors are orthogonal, then their dot product is 0. As the two vectors point more closely in the same direction, their dot product increases. The difference between the quadratic cost and linear reward functions in QP and LP is illustrated in Figure 2. The dot product between the desired and computed vectors closely matches the result of quadratic cost objective function.

The term \(C_u k\) relates to the velocity \(\dot{q}_{rn}\) that will be projected into the nullspace of the Jacobian, e.g., in order to move the arm away from joint limits. The objective function coefficient \(C_u\) is a parameter to LC\textsuperscript{3} indicating the relative importance of matching the desired acceleration \(\dot{X}_r\) compared to applying the nullspace projection of \(\dot{q}_{rn}\). The optimization variable \(k\) is the actual gain for the nullspace projection of \(\dot{q}_{rn}\). We additionally limit \(k\) to a maximum value with the constraint \(k \leq k_{max}\).

For some robot states \((q, \dot{q})\) and desired workspace velocities \(X_r\), the desired workspace acceleration \(\dot{X}_r\) may not be possible to achieve under the given kinematic constraints. For these cases, the result of the LP will include an optimal workspace acceleration \(\dot{X}_u\) that respects the constraints of the robot.

B. Derivation of Constraints

We derive linear constraints for position, velocity, and acceleration limits of the manipulator. The constraints, as with any LP, are a set of inequalities (1) based on the optimization variables, which for our problem are commanded acceleration \(\dot{X}_u\) and nullspace projection gain \(k\).

We can express all kinematic limits in terms of commanded change in velocity at each time step, \(\Delta \dot{q}_u\). We compute \(\Delta \dot{q}_u\) from commanded acceleration \(\dot{X}_u\) and nullspace projection gain \(k\):

\[
\Delta \dot{q}_u = J^+ \Delta t \dot{X}_u + k N \Delta \dot{q}_{rn}
\]

Equation (3) uses the Jacobian damped pseudoinverse \(J^+\) to transform a workspace velocity into a jointspace velocity vector so that we can apply joint-space constraints. The controlled acceleration, \(\dot{X}_u\), is multiplied by the current change in time \(\Delta t\) to get an instantaneous velocity change, i.e., an Euler integration step, resulting in \(\Delta \dot{q}_u\). The resultant velocity change is singularity robust because it uses the Jacobian damped pseudoinverse \(J^+\).

The last term in (3) is the projection of the change of the desired joint velocity \(\Delta \dot{q}_{rn}\) into the nullspace of the manipulator Jacobian \(N\). It is scaled by the optimization variable \(k\). We calculate the change in desired joint velocity \(\Delta \dot{q}_{rn}\) as the difference between the actual and desired joint velocities:

\[
\Delta \dot{q}_{rn} = \dot{q}_{rn} - \dot{q}_a
\]
Now, we derive the corresponding commanded joint velocities $q_u$ and accelerations $\dot{q}_u$.

From (3), the commanded joint velocity $\dot{q}_u$ is:

$$\dot{q}_u = \dot{q}_a + \Delta \dot{q}_u$$

Then, the corresponding commanded acceleration computed via finite difference is:

$$\ddot{q}_u = \frac{\dot{q}_u - \dot{q}_a}{\Delta t} = \frac{\Delta \dot{q}_u}{\Delta t}$$

Next, we write the constraints in terms of these commanded joint-space values.

1) Acceleration Constraints: Joint accelerations are limited by minimum $\ddot{q}_{\text{min}}$ and maximum $\ddot{q}_{\text{max}}$:

$$\ddot{q}_{\text{min}} \leq \ddot{q}_u \leq \ddot{q}_{\text{max}}$$

Replacing $\ddot{q}_u$ with the result from (5) and rearranging terms, we write the inequality in terms of $\Delta \dot{q}_u$ from (3):

$$\ddot{q}_{\text{min}} \Delta t \leq \Delta \dot{q}_u \leq \ddot{q}_{\text{max}} \Delta t$$

2) Velocity Constraints: Joint velocities are limited by minimum $\dot{q}_{\text{min}}$ and maximum $\dot{q}_{\text{max}}$:

$$\dot{q}_{\text{min}} \leq \dot{q}_u \leq \dot{q}_{\text{max}}$$

Again, we rearrange terms to write the inequality in terms of $\Delta \dot{q}_u$:

$$\dot{q}_{\text{min}} \Delta t \leq \Delta \dot{q}_u \leq \dot{q}_{\text{max}} \Delta t - \dot{q}_a$$

3) Position Constraints: We derive the position constraint based on distance traveled, $x$, for an initial velocity $\dot{x}_0$ and constant acceleration $\ddot{x}$ during time step $\Delta t$:

$$x = x_0 + \dot{x}_0 \Delta t + \frac{1}{2} \ddot{x} \Delta t^2$$

The manipulator’s joint positions are limited by minimum $q_{\text{min}}$ and maximum $q_{\text{max}}$. Given the current joint position $q_a$, the controlled joint velocity $\dot{q}_u$, and acceleration limits $\dot{q}_{\text{min}}$ and $\dot{q}_{\text{max}}$, we use (8) to find minimum and maximum achievable positions, $q_{\text{min}}$ and $q_{\text{max}}$, of the joint for the current control cycle:

$$q_{\text{max}} = q_a + \dot{q}_u \Delta t + \frac{1}{2} \ddot{q}_{\text{max}} \Delta t^2$$

$$q_{\text{min}} = q_a + \dot{q}_u \Delta t + \frac{1}{2} \ddot{q}_{\text{min}} \Delta t^2$$

If these exceed the respective bounding limits $q_{\text{min}}$ or $q_{\text{max}}$, then the controlled velocity $\dot{q}_u$ is invalid. This gives the bounds on the position:

$$q_{\text{min}} \leq q_{\text{max}}$$

Rearranging terms, we state the position limit in terms of $\Delta \dot{q}_u$:

$$\frac{q_{\text{min}} - q_a}{\Delta t} - \frac{\ddot{q}_{\text{max}} \Delta t}{2} - \dot{q}_a \leq \Delta \dot{q}_u \leq \frac{q_{\text{max}} - q_a}{\Delta t} - \frac{\ddot{q}_{\text{min}} \Delta t}{2} - \dot{q}_a$$

4) Simplified Combined Constraints: By stating all constraints in terms of $\Delta \dot{q}_u$, we can combine these into a single inequality, reducing the number of constraints by a factor of three. We find LP bounds from the most-constrained values in inequalities (6), (7), and (9).

$$c_{\text{min}} = \max \left\{ \frac{q_{\text{min}} - q_a}{\Delta t} - \frac{\ddot{q}_{\text{max}} \Delta t}{2} - \dot{q}_a, \frac{q_{\text{max}} - q_a}{\Delta t} - \frac{\ddot{q}_{\text{min}} \Delta t}{2} - \dot{q}_a \right\}$$

$$c_{\text{max}} = \min \left\{ \frac{q_{\text{min}} - q_a}{\Delta t} - \frac{\ddot{q}_{\text{max}} \Delta t}{2} - \dot{q}_a, \frac{q_{\text{max}} - q_a}{\Delta t} - \frac{\ddot{q}_{\text{min}} \Delta t}{2} - \dot{q}_a \right\}$$

Then, the resulting LP constraint is:

$$c_{\text{min}} \leq \Delta \dot{q}_u \leq c_{\text{max}}$$

C. Sign Transformation of Optimization Variables

The optimization variables in standard LP form must be positive. However, cases where the robot’s controlled velocity cannot match the sign of the desired velocity, e.g., due to a large initial velocity in the opposing direction, must also be handled. This restriction is relaxed as we are operating over accelerations in workspace, and the sign of the desired acceleration must be matched. To account for this, we create sign transformation matrix $M$ to convert the optimization variables to positive values. $M$ is a diagonal matrix whose entries are the sign of the desired workspace acceleration $\ddot{X}_r$:

$$M_{ij} = \begin{cases} 0 & i \neq j \\ \text{sign}(\ddot{X}_r) & i = j \end{cases}$$

In the case of the workspace reference acceleration $\ddot{X}_r$ having 0 for an entry, the 0 is treated as a positive variable so $M$ is always invertible.

We apply $M$ to matrices $J^+$ and $\ddot{X}_u$ to ensure positive optimization variables:

$$\ddot{J} = J^+ M^{-1} \ddot{X}_u = M \ddot{X}_u$$

As $M^{-1} M$ equals the identity matrix, the result of multiplication is preserved:

$$\ddot{J} \ddot{X}_u = J^+ M^{-1} M \ddot{X}_u = J^+ \ddot{X}_u$$

Consequently, the LP objective function from Equation 2 is:

$$\ddot{X}_r \cdot \ddot{X}_u + C_u k$$

Then, Equation 3 becomes:

$$\Delta \dot{q}_u = \ddot{J} \Delta t \ddot{X}_u + N \Delta \dot{q}_{\text{frn}} k$$

Using the sign transformation matrix $M$ ensures that optimization variable vector $[\ddot{X}_u, k]$ will be positive. We can then recover the actual value for commanded acceleration via $M^{-1}$:

$$\ddot{X}_u = M^{-1} \ddot{X}_u$$
D. Standard LP Form

We now represent $LC^3$ in the standard LP form given in (1). Writing our problem in this canonical form enables use of efficient algorithms [4] and solvers [3].

The objective function for optimization is:

$$\vec{X}_r \cdot \vec{X}_u + C_u k$$

The constraints are:

$$c_{\min} \leq \vec{J}^+ \Delta \vec{X}_u + N \Delta \vec{q}_r n k \leq c_{\max}$$

$$k \leq k_{\max} \quad \vec{X}_u \leq \vec{X}_r$$

The manipulator and LP values are of the following dimensions:

- Joint-space vector $q \in \mathbb{R}^n$
- Workspace vector $X \in \mathbb{R}^m$
- LP optimization variable vector $x \in \mathbb{R}^{m+1}$
- LP objective function vector $c \in \mathbb{R}^{m+1}$
- LP bounding vector $b \in \mathbb{R}^{2n+m+1}$
- LP constraint matrix $A \in \mathbb{R}^{(2n+m+1) \times (m+1)}$

The LP vectors $x$, $b$, and $c$ in Equation 1 are:

$$x = \begin{bmatrix} \vec{X}_u \\ k \end{bmatrix}, \quad c = \begin{bmatrix} \vec{X}_r \\ C_u \end{bmatrix}, \quad b = \begin{bmatrix} c_{\max} \\ -c_{\min} \\ k_{\max} \\ \vec{X}_r \end{bmatrix}$$

The LP constraint matrix $A$ is:

$$A = \begin{bmatrix} \vec{J}^+ \Delta t & N \Delta \vec{q}_r n \\ -\vec{J}^+ \Delta t & -N \Delta \vec{q}_r n \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

All together, the LP is as follows:

$$c^T x = \begin{bmatrix} \vec{X}_u \\ C_u \end{bmatrix} \begin{bmatrix} \vec{X}_u \\ k \end{bmatrix} = \vec{X}_r \cdot \vec{X}_u + C_u k$$

$$Ax \leq b \rightarrow \begin{bmatrix} \vec{J}^+ \Delta t & N \Delta \vec{q}_r n \\ -\vec{J}^+ \Delta t & -N \Delta \vec{q}_r n \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \vec{X}_u \\ k \end{bmatrix} \leq \begin{bmatrix} c_{\max} \\ -c_{\min} \\ k_{\max} \\ \vec{X}_r \end{bmatrix}$$

III. Experimental Results

We implement and demonstrate $LC^3$ on simulated and physical robot manipulators. Our implementation of $LC^3$ is written in C using openBLAS [13], [26] for linear algebra and lp_solve [3] for linear programming. We test our software on an Intel® i7-4790 CPU under Linux 3.18.16-rt13+ PREEMPT RT. Figure 3 summarizes the computational performance of $LC^3$ compared to the baseline Jacobian damped least-squares methods (DLS).

We compare a baseline Jacobian DLS with $LC^3$ by servoing the manipulator to a desired workspace position. The desired workspace velocity is the logarithm of error between actual pose $B_S_c$ and desired pose $B_S_r$:

$$\vec{X}_r = \begin{bmatrix} \omega_r \\ v_r \end{bmatrix} = \ln \left( B_{S_c}^* \otimes B_{S_r} \right)$$

![Fig. 3. Computation time results for $LC^3$ compared to baseline Jacobian Damped Least Squares (DLS) for a 7 DOF arm. We compared the Jacobian DLS and $LC^3$ using both the LU decomposition and the SVD to compute the Jacobian damped pseudo-inverse. The CPU was an Intel i7-4790. All methods are fast enough to operate at typical control rates of 1 kHz with $LC^3$ taking about 20 times as long as the baseline due to solving the linear program in addition to computing the damped pseudo-inverse.](image-url)

![Fig. 4. Simulation on the UR10 robot, maneuvering its end-effector to a point. The beginning joint configuration has all of the robot’s joints at their zeroed position. The top row shows the joint positions $q_a$, and the bottom row shows the controlled actual joint velocities $\dot{q}_a$. Each line represents one joint on the manipulator. The baseline DLS has a large velocity spike at the beginning of execution (c), while $LC^3$ has a gradual increase, respecting the acceleration limits on the system (d). Both converge to the desired position in approximately the same time.](image-url)

![Fig. 5. Images of simulated UR10 robot. The arm on the left is using $LC^3$, and the arm on the right is using Jacobian DLS (JDLS). (a) shows the larger initial displacement of the right manipulator due to the large initial velocity spike of the Jacobian DLS. (b) shows how both $LC^3$ and Jacobian DLS converge to approximately the same configuration at the same time.](image-url)
where $\omega_r$ is reference rotational velocity, $\dot{\omega}_r$ is reference translational velocity, $B_S^i$ is the actual end-effector pose dual quaternion, $B_{S'}^i$ is the reference pose dual quaternion, and $\otimes$ is the quaternion multiplication operator.

We use kinematic redundancy to center the joints within the middle of their position ranges, thereby avoiding poor manipulator configurations which may prevent movement. We compute the nullspace projection velocity $\dot{q}_{rn}$ to center each joint as:

$$
(\dot{q}_{rn})_i = \frac{(q_c)_i - (q_a)_i}{(q_{max})_i - (q_{min})_i}
$$

where $(\dot{q}_{rn})_i$ is the $i^{th}$ element of the nullspace projection velocity, $(q_c)_i$ is the $i^{th}$ joint center position, $(q_a)_i$ is the $i^{th}$ actual velocity, and $(q_{max})_i$ and $(q_{min})_i$ are the $i^{th}$ joint position limits.

### A. Simulation Results

We present kinematic simulation results on the Universal Robotics UR10 and Rethink Robotics Baxter robots. The UR10 is a 6-DOF non-redundant manipulator, while the arms of the Baxter are 7-DOF redundant, anthropomorphic limbs. We simulate servoing to a point in workspace from a nominal starting configuration, comparing LC$^3$ with the baseline Jacobian damped least-squares approach. The positions and velocities of the UR10 robot are plotted in Figure 4 and those of the Baxter in Figure 6. There is a dramatic difference in velocities of the UR10 robot are plotted in Figure 4 and those of the Baxter when its arms have been untucked. The format of the plots is the same as Figure 4. Similar profiles as the other simulation can be seen. As the Baxter’s manipulators are redundant, the velocity $\dot{q}_{rn}$ is projected into the nullspace of the Jacobian.
difference in initial velocity, both $LC^3$ and the baseline converge to the final joint positions at approximately the same time. These results show that $LC^3$ provides a smooth velocity ramp for redundant and non-redundant manipulators.

B. Physical Robot Results

We demonstrate $LC^3$ on a physical Baxter robot. As in the simulation, we compare $LC^3$ and the baseline Jacobian DLS for servoing to a workspace position. The actual joint positions and velocities $q_a$ and $\dot{q}_a$ along with the commanded velocities $\dot{q}_a$ are shown in Figure 8. Note the difference between the commanded and actual velocities, arising due to physical limits of the manipulator. Just as in the simulation case, $LC^3$ provides reduced acceleration and velocity requirements while converging in similar time compared to the baseline.

IV. DISCUSSION AND CONCLUSION

We presented a new Cartesian workspace controller, linearly-constrained Cartesian control ($LC^3$). This new method respects the position, velocity, and acceleration constraints in the manipulator’s joint-space. $LC^3$ is singularity-robust, provides smoother and more gradual motions of the manipulator, and in the case of redundant manipulators, maneuvers the arm away from poor configurations near joint limits that reduce maneuverability. We demonstrated this controller in simulation and on a physical manipulator.

There are several tuneable parameters within $LC^3$. These include the damping constant, $\lambda$ or $s_\alpha$, for the Jacobian damped pseudo-inverse $J^+$, the weighting constant of the nullspace projection in optimization $C_{uv}$, the maximum possible value of the nullspace projection gain $s_{max}$, and the constraints upon the joints themselves. These all can affect the performance of the controller in terms of achievability and accuracy. Furthermore, it is possible to modify the weighting of joints in the nullspace projection velocity to change the resulting motion. For example, by more-heavily weighting joints with smaller range of motion, the projection favors centering these joints instead of those with greater range of motion. Generally this helps the manipulator remain in configurations with greater reachability.

A key advantage of $LC^3$ is increased robustness to initial state compared to the baseline Jacobian damped least-squares (DLS). The DLS may produce large accelerations from some initial configurations or require additional velocity ramping to produce smooth motion, and it does not consider initial velocity. In contrast, $LC^3$ will produce acceleration-limited motion regardless of initial configurations or velocities.

REFERENCES


